A Theory of Incomplete Measurements

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Abstract

Physics is not just about mathematics. Mathematical entities in physics translate an actual experience. Measurements bridge this gap. A definition of measurements in six common sense postulates offers an interesting and possibly new insight into the structure of quantum mechanics, general relativity, and on techniques such as renormalization.

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1 Introduction

For a very long time, we have used mathematics to represent the laws of physics. For instance, we write Newton's second law as $\vec{F} = m\,\vec{a}$, where $\vec{F}$ and $\vec{a}$ are "vectors", abstract mathematical entities made up of 3 real numbers, which are themselves abstract mathematical entities. The mathematics being used has only grown more sophisticated with time, to the point where, in recent days, progress in mathematics and progress in physics are often simultaneous. This use of mathematics, and the success we had with it, is so essential to any kind of modern physics that we practically never question it.

The present article studies how and why mathematics can actually be used to represent the physical world. The fundamental premise is so simple that it is generally overlooked entirely: the mathematical entities we manipulate are symbolic representations for the result of measurements. For example, $\vec{F}$ represents a physical force, something that we can measure for instance with a dynamometer; m represents a mass, that we can measure with weights; $\vec{a}$ is an acceleration, that we can derive from speed or position measurements; and so on.

However, a question follows from the fundamental premise: does it matter, how we measure $\vec{F}$, m or $\vec{a}$? The laws of physics are never written with a specification of how you must measure a given physical entity. Therefore, the classical answer appears to be that it does not matter.

The main thesis presented here is, on the contrary, that the physical measurement being chosen does matter. Furthermore, we will attempt to demonstrate that many of the properties that we take for granted, or that we use as axioms for physics laws, are indeed a consequence of the choices we make with respect to the physical processes we accept to call "measurements".

In the process, we will establish a number of essential properties of both general relativity and quantum mechanics, which become two approximations of the theory presented here. It is important to note that this result is not obtained by suggesting some new and improved Lagrangian or action that models both, but by revisiting a much deeper foundation of physics theories, namely the very meaning given to variables like x in the equations.

Overview In the first section of this article, we will propose a definition of measurements (section 2.1), introduce a notation allowing us to reason about this and other physics problem without constraining the mathematics (section 2.2), and illustrate how the formalism can be used to discuss simple problems (section 2.3).

The next section will focus on general relativity, beginning with a definition of space and time based on measurements (section 3.1). We will explain special relativity as a transcription of observed relations between multiple measurements of distance (section 3.2). We will argue that the use of curved space-time representations in general relativity is justified because we do not know how to build an Euclidean geometry out of physical measurements (section 3.3). We will distinguish among the equations of general relativity those that originate in mathematics or geometry, and those that derive from physical observations (section 3.4). Finally, we will show that the relatively recent idea of scale relativity may be justified when we change measurements to "zoom in" or "zoom out" (section 3.5).

The last section will focus on quantum mechanics. We will first demonstrate that quantum mechanics axioms can be derived from the existence of measurements that can only be predicted with probabilities (section 4.1). We will illustrate the precise mathematical parallel with the traditional formalism, justify the emergence of a complex-valued wave-function describing the probability of presence, and suggest a discrete (as opposed to continuous) normalization condition for the wave-function (section 4.2). This will lead us to a new interpretation of quantum mechanics devoid of the so-called "quantum measurement problem" (section 4.3).

2 Defining Measurements

While we live in the universe, we have no direct perception of it. Instead, we rely on measurements to gather information about the universe, whether performed by biological physical processes in our body or by physical devices we invented. All physical laws hinge on measurements, which suggests that a precise definition can give us new insights on the meaning and validity of these physical laws.

2.1 Measurement Postulates

This article postulates that a measurement is:

1. a valid physical process,
2. impacting two fragments[1] of the universe chosen in advance,
3. in a repeatable way,
4. gathering information about an *unknown* (or *input*) fragment
5. represented by a change in a *display* (or *output*) fragment
6. which can be given a numerical or symbolic interpretation.

It may be useful to justify each of these propositions.

1. A measurement is done from within the universe, and it obeys the laws of the universe. It must therefore be a valid physical process. This proposition states that a voltmeter does not operate through magic, but through the normal laws of physics.
2. In order for a measurement to be useful, it needs to correlate fragments of the universe that are known to the measurement observer. A voltmeter must measure its inputs, not a voltage picked up at random in the universe, and it must show the result in a known way on a previously identified display.
3. A measurement needs to be repeatable, giving consistent results for consistent inputs. A voltmeter that, given the same input, displays random values is considered unreliable, and it should not be used for actual measurements.
4. A measurement needs to gather information about the unknown fragment. When you roll a dice, this is a valid physical process, and it presumably correlates the final value shown by the dice with the initial position and speed of your hand. However, it does not present useful information about the state of your hand, and therefore is not considered a measurement.
5. Since we live in the universe, the result of the measurement manifests as a change in the "display" universe fragment. If your voltmeter display does not move in response to voltage, or moves noticeably in response to some other influence, the voltmeter is broken and cannot be used to perform actual measurements.
6. In order for the measurement to give quantitative and not just qualitative results, the "display" fragment of the universe is associated with a graduation, which allows us to map the changes to a symbolic or numeric value. Only this graduation and associated calibration allow us to know that a particular voltmeter state corresponds to a particular voltage on the inputs.

1. The term *fragment* is used here to indicate that we do not assume any specific partitioning of the universe along any particular variable. Except for this distinction, it is otherwise essentially used in the same sense as *system* or *region of space-time* have been used in many physics texts.

These statements are a possible definition of what we want to accept as measurements, and not a statement about how the universe works. We can however make a statement about the universe: experimentally, there are physical processes with these properties. In particular, repeatability relies on the observed existence of symmetries, such as invariance by rotation or translation through space and time, allowing multiple identical physical processes in the universe.

2.2 Formal transcription

It is possible to transpose the statements made earlier into a formal notation that facilitates a precise description of complex statements about the universe.

Possible Let us denote $\mathfrak{F}$ the fragment of the universe $\mathfrak{U}$ on which a measurement M is being performed (the measurement apparatus input), $\mathfrak{D}$ the display fragment where the result shows up (the measurement apparatus output), and P an arbitrary physical process that is not necessarily a measurement.

We can define the notation $P\,\mathfrak{F}$ as the application of process P to universe fragment $\mathfrak{F}$. In reality, P may apply to the universe as a whole, but the notation $P\,\mathfrak{F}$ tells us that:

- the physical process P is compatible with the state of the universe fragment $\mathfrak{F}$.
- we are interested in the effects of the process P with respect to the fragment $\mathfrak{F}$. This does not necessarily mean that the effect of P is restricted to $\mathfrak{F}$, as discussed below.

The first point shows that the notation can be used as a predicate, telling the reader that a physical process is valid for the universe fragment under consideration. The second point indicates that the notation can also be used as an operator notation, allowing use to denote the changes in the universe that are a consequence of the application of the physical process P.

The notation $M\,\mathfrak{F}$ expresses the first measurement postulate, since it reads as "*M is a physical process that applies to $\mathfrak{F}$*".

Correlating We can define the notation $\mathfrak{F}_1 = \mathfrak{F}_2$ to indicate that what we know about the universe fragment $\mathfrak{F}_1$ is consistent with what we know about the universe fragment $\mathfrak{F}_2$ and conversely. A notation using an equal sign is reasonable, because the relation is reflexive, symmetric and transitive like the traditional mathematical equality.

- Reflexivity is necessary for logical consistency of our knowledge of the world, as what we know about a system must not be self-contradictory.
- Since the relation denotes consistency between what we know about two fragments of the universe, it is symmetric by construction.
- Transitivity is also required for a logically consistent system of knowledge applied to any three fragments $\mathfrak{F}_1$, $\mathfrak{F}_2$ and $\mathfrak{F}_3$. If what is known about $\mathfrak{F}_1$ is compatible with what is known about both $\mathfrak{F}_2$ and $\mathfrak{F}_3$, then there can be no incompatibility between what is known about $\mathfrak{F}_2$ and $\mathfrak{F}_3$, as any such known incompatibility would imply an incompatibility with $\mathfrak{F}_1$

However, the relation $\mathfrak{F}_1 = \mathfrak{F}_2$ does not necessarily map a single state of the universe fragment $\mathfrak{F}_1$ to a single state of the universe fragment $\mathfrak{F}_2$. It is for example possible for this equation to hold when we know nothing about either $\mathfrak{F}_1$ or $\mathfrak{F}_2$. Any such equality relationship also requires various physical hypotheses to be valid, and implicitly carries these hypotheses with it. It would be very tedious to make this explicit in the notation, but one should always be thinking that, strictly speaking, $\mathfrak{F}_1 = \mathfrak{F}_2$ stands for $\mathfrak{F}_1 =_H \mathfrak{F}_2$, where H is an index representing the set of physical hypotheses under which the equality is being considered. One particularly notable hypothesis is the approximation consisting in ignoring irrelevant aspects of the system.

Based on the definition of equality above, we can then write eq. (1) to indicate that a measurement M is a valid physical process collecting information from a universe fragment $\mathfrak{F}$ and showing that information in the "display" fragment $\mathfrak{D}$:

$$M\,\mathfrak{F} = M\,\mathfrak{D} \tag{1}$$

It may be helpful to suggest that eq. (1) should read as: *The information we have*[2] *about the display fragment* $\mathfrak{D}$ *after applying measurement* M *is consistent with the information we have about the unknown fragment* $\mathfrak{F}$ *after applying the same measurement* M.

In particular, eq. (1) does not specify anything about the rest of the universe, and specifically nothing about $\mathfrak{F}$ and $\mathfrak{D}$ "before" the measurement. Note that both sides of eq. (1) contain M because we are interested in the effect on the fragments $\mathfrak{F}$ and $\mathfrak{D}$ of the same physical process M. An equation like $\mathfrak{D} = M\,\mathfrak{F}$ would indicate that applying the process M to a fragment $\mathfrak{F}$ transforms it into $\mathfrak{D}$, which is not what happens in a measurement. By contrast, what eq. (1) tells us is only that we can choose $\mathfrak{F}$ and $\mathfrak{D}$ "before" performing the measurement, which is the second measurement postulate.

Repeatable The measurement process needs to be repeatable. After applying the measurement once, applying it a second time should result in the same observation. Without invoking any notion of "time", we can write eq. (2), which holds for measurements but not for arbitrary physical processes:

$$M\,M\,\mathfrak{F} = M\,\mathfrak{F} \tag{2}$$

Based on the definitions given earlier, eq. (2) reads as "*what we learn about the system* $\mathfrak{F}$ *by applying the measurement* M *is compatible with what we learn about* $\mathfrak{F}$ *by applying* M *to the fragment of the universe that resulted from the measurement process*", a more precise definition of the third measurement postulate.

Focused The measurement process correlates two selected fragments of the universe $\mathfrak{F}$ and $\mathfrak{D}$, but it really collects information only about $\mathfrak{F}$, something which is not apparent in eq. (1): it does not depend on the rest of the universe. This is described by eq. (3):

$$M\,\mathfrak{F} = M\,\mathfrak{U} \tag{3}$$

At first glance, eq. (3) might appear "dimensionally challenged" if one does not remember that, as above, it relates what we know about $\mathfrak{F}$ and what we know about $\mathfrak{U}$. A possible reading of that equation would be: "*all that we learned about the universe* $\mathfrak{U}$ *by applying measurement* M *is what we learned about the unknown fragment* $\mathfrak{F}$ *by applying measurement* M". The measurement process M can give information only about $\mathfrak{F}$, which is the fourth measurement postulate.

As indicated earlier, other physical processes may have this property. Such processes will be said to be *focused on* $\mathfrak{F}$.

Observable The measurement process correlates an input and an output as shown in eq. (1), but the relation appears symmetric. In reality, we only observe the output of the measurement apparatus. Let us define the notation $|\mathfrak{D}|$ to indicate the objective interpretation of the changes in the display fragment $\mathfrak{D}$, in other words an interpretation of the changes that does not depend on the observer making it, nor even requires an observer. We can read the notation $|\mathfrak{D}|$ as "*the output fragment* $\mathfrak{D}$ *is directly observable*[3] *with measure* $|\mathfrak{D}|$". This notation expresses our fifth measurement postulate.

It is very possible that for some experiments, we can only observe the measurement output directly, but not the measurement input. In that case, the notation $|\mathfrak{D}|$ is acceptable but the notation $|\mathfrak{F}|$ is not valid.

Quantifiable For M to be a measurement, it must also be possible to give a numerical or symbolic interpretation m to the changes that are caused by the physical process M to the display $\mathfrak{D}$. We can use a "*graduation equation*" like eq. (4) to show how a particular state of the display fragment $\mathfrak{D}$ is interpreted numerically.

$$|M\,\mathfrak{D}| = m \tag{4}$$

2. We must particularly emphasize the phrase "*the information we have*" in the definition of the "equality" found in eq. (1). This equality is not intended to indicate perfect identity between two physical systems, something which, in the current state of our knowledge, may not make sense. On the contrary, such an equation implicitly takes into consideration all the imperfections and limitations of our measurement systems and theory. This approach is not unreasonable, and can be related to the way physicists today routinely write perfect equalities like $\vec{F} = m\,\vec{a}$ with a clear understanding of the various approximations behind this notation.

3. The word *observable* is used here in the original English sense, not in the derived quantum-mechanical acceptation of the term. Furthermore, the word *observable*, contrary to *observed,* does not imply an actual observer, but only that an objective interpretation of the change in $\mathfrak{D}$ is possible.

Per the definitions above, $|M\,\mathfrak{D}|$ denotes the directly observable change in the display fragment that results from the measurement process. We can read eq. (4) as "*the changes in the display fragment that occur during measurement are associated to the symbolic measure m*", a transcription of the sixth measurement postulate.

Graduations are somewhat arbitrary. They integrate the frequently used notion of measurement *unit*. A graduation like eq. (4) relates the possible states of $\mathfrak{D}$ to the possible values of m. We do not specify the mathematical formulation of m, and specifically, m does not need to be a real number, or even a number at all. For example, eq. (4) could relate an unknown or partially known state of $\mathfrak{D}$ to a probability distribution m. However, in eq. (4) unlike in earlier equations, the equal sign is used in the more traditional meaning of equality between mathematical entities such as real numbers or vectors.

2.3 Formal reasoning

If we stay within physically reasonable limits, we can use the equations above to do some reasoning about the results of a measurement. In this section, we will give a few examples, not with the intent to be exhaustive, but for illustration purpose.

Repeatable Process In many cases, a same process P can be repeated a large number of times. If we apply the process P a total of k times to a universe fragment $\mathfrak{F}$, we can write $P\,P\,P\,...\,P\,\mathfrak{F}$ as $P^k\,\mathfrak{F}$. In that case, $P^k\,\mathfrak{F}$ it is to be understood as being valid only for physically acceptable values of P, k and $\mathfrak{F}$. Such a process P is said to be *repeatable*.

A particular application of this notation is when the physical process can somehow be reversed as far as the system being studied is concerned. For example, when observing the layout of a room, the physical process reversing "adding a chair to the room" is the process "removing the chair that was added". Such a process is said to be *reversible*. Note that whenever an inverse is used, the approximations required for this inversion implicitly become part of the set of hypotheses H used in the definition of the equality. For example, the inverse "removing the chair that was added" requires that one ignores aspects that are not known to be identical, such as the position of the molecules of air in the room.

If such a process exists for process A, we can denote A^{-1} the inverse process, which obeys eq. (5) for all physically acceptable value of A, k and $\mathfrak{F}$. In particular, depending on the physical process being considered, this may include the cases $k=0$ or $k<0$.

$$A^{-1}\,A^k\,\mathfrak{F} = A^{k-1}\,\mathfrak{F} \tag{5}$$

At a macroscopic scale, most measurement processes appear to have, at least approximately, an inverse with respect to the system being measured and the measurement process. For example, the inverse of "connecting a voltmeter" is "disconnecting a voltmeter"; the inverse of "placing a rod alongside an object" is "putting the rod back in the original place". This is not a general rule: destructive measurements do not have an obvious inverse, since the object being measured cannot easily be reconstructed.

Identical Processes In many cases, we will have to write equations that are valid for all physically acceptable systems, such as eq. (5). A shortcut notation in that case is to consider this an identity of the physical processes themselves, as in eq. (6):

$$A^{-1}\,A^k = A^{k-1} \tag{6}$$

This equation reads as follows: "*for all acceptable physical systems, what we know about the system when applying the physical process $A^{-1}A^k$ is compatible with what we know about the system when applying the physical process A^{k-1}*".

When an identity $P = Q$ holds, the processes P and Q are said to be *identical*. In practice, two processes may be identical only for a large range of fragments $\mathfrak{F}$, in which case the identity $P=Q$ is an approximation valid on that range.

Quasi-identical Processes In general, two physical processes P and Q are not truly identical as far as we know, but we can find a reversible physical process T so that $P=TQ$. In that case, we will say that P and Q are identical by transformation T. In particular, we will often consider physical translation or rotation transformations, and state that P and Q are identical by rotation or translation.

While it is more precise to say that two physical processes are *quasi-identical* if they are identical by a combination of translations and rotations through space and time, in the rest of this article, we will simply call them identical unless the context requires the clarification. Furthermore, since we based all our definitions of "identical" on what we know about $\mathfrak{F}$ or A, it is actually possible for distinct physical systems to be truly identical[4] in that sense.

Commutativity Consider now two independent measurements X and Y, "*independent*" meaning that the result of X does not depend on the fact that we performed a measurement of Y previously, and conversely. The first condition can be written as eq. (7) and the second condition can be written as eq. (8):

$$X = XY \tag{7}$$

$$Y = YX \tag{8}$$

If we further know an inverse process X^{-1}, then eq. (8) yields $XY = XYX$, eq. (6) leads to $XY = X^{-1}XXYX$, eq. (2) leads to $XY = X^{-1}XYX$. Finally, another application of eq. (6) results in eq. (9):

$$XY = YX \tag{9}$$

In physical terms, this shows that under the conditions stated above, two physical measurements that are independent from one another correspond to commuting physical processes. This result is particularly interesting in relationship with quantum mechanics, where such commutation relations are the foundation for the Heisenberg uncertainty principle[1, 2, 3]. After comparatively very little work, it is satisfying to find a result which appears qualitatively compatible with validated theories and experiments. We will return to this topic later.

Note that eq. (7) and eq. (8) also hold when X and Y are the same measurement. Using eq. (2), one could argue that a measurement process is also independent from itself, since its result does not change if it was performed earlier.

Nonperturbing behavior Another formal manipulation is that $Y = (X^{-1}X)Y$ by definition of the inverse, and then since $XY = X$ this means that $Y = X^{-1}X$. In other words, Y appears to be indistinguishable from an "identity" process.

Naturally, one should certainly not interpret that as indicating that it is an identity process respective to any arbitrary physical process, but only with respect to the processes being considered. The corresponding approximations are part of the set of hypotheses H required for the "equality" to hold. In particular, we do not know if X or Y commutes with any other physical process or if the inverse processes we chose for X or Y are legitimate inverses for any other process. Consequently, without additional hypotheses, we cannot deduce from this reasoning anything about the interaction of X and Y with any other physical process.

However, one can say that for the conditions listed above to hold, X and Y must behave as an identity, i.e. they do not perturb the system as far as either X or Y is concerned. This condition corresponds to the "ideal measurement" in classical mechanics, where the conditions listed above generally hold. In other words, measurements in classical mechanics that commute and have an inverse can be considered as not perturbing the system in any way. A related observation is that, in that case, $|Y\mathfrak{F}| = |\mathfrak{F}|$: the result of the measurement does not require that the measurement be actually performed. Again, this corresponds to the common classical mechanics "reality hypothesis" that the value of a measurement predates the actual measurement.

Linearity If A is a repeatable process, $A\,\mathfrak{F}$, $A^2\,\mathfrak{F}$, and more generally $A^k\mathfrak{F}$ all describe valid physical processes applied to $\mathfrak{F}$. Experience tells us that sometimes, a measurement M is unable to distinguish between one instance of A and another. For instance, if A is a process adding an identical photon each time, measurements will not be able to tell one of the added photons from another.

4. Physics texts often use the word *indistinguishable* in that case.

When these conditions are met, we can choose for M a *linear graduation* obeying eq. (10): even if the measurement cannot distinguish between two individual instances of the process A, it may still help us count how many times it occurred.

$$|M A^n \mathfrak{F}| = n\ |M A \mathfrak{F}| \tag{10}$$

The precise meaning of the multiplication in eq. (10) depends on the actual mathematical object used to represent $|M\ \mathfrak{F}|$ for this particular measurement. However, the general structure that emerges from these relations, linearity, does not depend on this choice.

As a matter of fact, the choice of linear graduation is never mandatory. If evolution had not given us a sense of vision, we could very well have decided to base our units of length not on counts of optically aligned identical reference objects, but on the energy it takes to throw an object at a given distance. This particular measure of distance would not be linear relative to ours. In practice, however, a linear graduation is often chosen, because the symbolic measures $|M A^n \mathfrak{F}|$ naturally follow a simple group structure.

It is actually not unreasonable to speculate that our species learned how to count trying to measure groups of identical objects. In other words, eq. (10) can be seen as the kind of relation from which we learned how to count. To fulfill the need to measure groups of pebbles, the simplest way is to consider all pebbles equivalent, in other words to make them "indistinguishable" for the measurement being considered. Then, arithmetic on natural numbers is the group structure that can be derived from the impact on this particular physical measurement of operations on groups of pebbles such as joining them (addition) and joining multiple groups with similar measure (multiplication).

Sometimes, a measurement may distinguish between two physical processes A and B without being able to distinguish two instances of A or two instances of B. If arbitrary sequences like $A\,B\,B\,B\,A\,B\,A\,A\,\mathfrak{F}$ can represent a physical process, and if physical processes A and B commute, we can extend the reasoning leading to eq. (10): if we cannot tell two instances of A apart, nor tell two instances of B apart, the best we can do is to count how many times each was applied.

More generally, if $\mathfrak{F}$ can be constructed from a "ground state" $\mathfrak{F}_0$ by repeated application of commuting physical processes $P_1, ..., P_n$, that is if $\mathfrak{F} = P_1^{k_1}\, P_2^{k_2} ... P_n^{k_n} \mathfrak{F}_0$, and if M is a measurement that cannot distinguish one P_i from another (even if it can distinguish a P_i from a P_j when $i \neq j$), then the graduation for M can be chosen to verify eq. (11):

$$m = |M\,\mathfrak{F}| = \sum_i k_i |M\,P_i\,\mathfrak{F}_0| = \sum_i k_i\, m_i \tag{11}$$

We can consider the physical states that are constructed from a common ground state $\mathfrak{F}_0$ using repeated applications of mutually independent physical processes P_i. As demonstrated earlier, the condition that the P_i are independent is sufficient for them to commute. Consequently, eq. (11) applies. By this definition, any such physical state $\mathfrak{F}$ can be written as $\mathfrak{F} = \prod\ P_i^{k_i}\,\mathfrak{F}_0$ and is therefore defined by the vector $(k_i) \in \mathbb{N}^n$. Under the condition identified above, measurements are linear operators on these vectors. This condition will be called the *linear approximation*, and it is frequently applicable in practice. For example, the measurements of mass for particles that move slowly with respect to one another follow a linear approximation. We will see shortly that this condition also appears to hold for measurements of distance and time.

2.4 Remarks

The formalism shown above allows us to reason quickly and efficiently about physical processes and measurements, and to define what we choose to call a measurement. However, it does not impose a particular mathematical formalism, such as Hilbert spaces or tensors. While it uses a mathematical notation of its own, it still does not really constrain the mathematical formulation of a theory of physics. For instance, the mathematical entity that we wrote as m in eq. (4) can be practically anything, from a simple binary value ("true", "false") to a much more complex way to represent the knowledge we gained about the physical system $\mathfrak{F}$ from measurement M.

Incompleteness In the rest of this article, we will refer to this formalism as the "theory of incomplete measurements" (TIM). There are two reasons for this name:

- Measurements are incomplete in the sense that they only are something *we know* about the universe, and not what the universe *is*.
- Measurements need not be instantaneous, and the formulation is uniform irrespective of how complete a particular measurement is.

More structure is required Additional observations must be made to give more structure to our knowledge of the universe and to build a theory with predictive power. Much of this knowledge can be borrowed from established theories. As a matter of fact, it probably must, since any new theory must obviously be compatible with what we already know. The following sections attempt to show the general direction that this effort should take.

Symmetry between all measurements In the TIM, no measurement, not even space or time, is given any particular pre-eminence over the others. This is in contrast with the most common formulations of both general relativity and quantum mechanics, where space and time measurements are generally given a very special role, space and time being the "background" where other elements of the theory are defined. Preserving the original symmetry while we discuss space and time will be an important constraint.

Non-equivalence of all measurements When one of the existing theories uses a variable, for instance x, that variable is implicitly associated with a physical measurement X. To the extent that the precise physical measurement X for x is never specified in the theory, one or several of the following assumptions is generally made, more or less implicitly:

- All physical measurements of x are equivalent as far as the theory is concerned.
- Differences between two physical processes X_1 and X_2 implementing a particular measurement x are of no concern for the mathematical properties of x.
- The measurement with the highest resolution or widest range is necessarily superior.

Using various examples, we will demonstrate later in this article that these assumptions are not justified. We can, however, give a quick analogy that helps understanding, intuitively, the possible negative effect of these assumptions. Measurements can, in a sense, be seen as functions mapping a system in the universe onto symbolic or numerical values. Two measurements agree when measuring a same object because they are *calibrated* to match.

One often overlooked but capital point is that the number of calibration systems is finite. Without additional knowledge, it is not more reasonable to assume that the two measurements are equal because they match on a finite number of calibration systems than it is reasonable to deduce the equality of two arbitrary mathematical functions based on their equality for a finite number of values.

Relativity of Measurements Special relativity asked us to give up the notion of "absolute speed", general relativity the notion of "absolute coordinates". It may be time to more generally give up the notion of "absolute measurement".

If we must abandon the assumption that the precise physical process for a given measurement has no bearing on the mathematical properties associated with the measurement results, we gain a new principle of relativity: *No measurement is by its nature better than another to express the laws of physics.*

A new principle of relativity makes it necessary to specify additional information in the theory. In special relativity, time is no longer absolute, so we need to specify what observer a time is relative to. In general relativity, no space-time coordinate system is naturally preferred, which means that we must specify a metric. In the TIM, no physical measurement is naturally preferred, which means that we must specify the physical measurement process.

3 Relativity Theories

The TIM formalism presented above allows us to take a fresh look at special and general relativity theories[4, 5], starting with the mathematical structure of space-time coordinates. The justification for this exercise is to identify, in the mathematical structure of these well-established theories, where the details of physical measurement processes may matter.

3.1 Space-time

The space-time coordinates are traditionally noted as x, y, z and t. In the formalism presented here, we need to specify how these coordinates are being measured.

Time Time is usually measured by counting the repetitions of a given physical phenomenon[6], whether it is a cosmological physical process like the rotation of earth around its axis, the oscillation of a pendulum, the oscillation of a quartz crystal, or the decay of the excited states of some atom in atomic clocks.

These measurements of time are therefore based on the repetition of a same physical process, the "beat" of the clock, that we can denote as δT, and on a distinct time measurement process T that counts these processes to measure a time. The general definition of a time measurement based on periodic processes is given in eq. (12), where linearity is a consequence from the fact that two δT are identical, or at least indistinguishable through measurement T. In this equation and the following, δT is the name of a process distinct from T, not some operator δ applied to T, so that the notation δT^k unambiguously means $(\delta T)^k$.

$$t = |T \delta T^k \mathfrak{F}_0| = k\,|T \delta T\, \mathfrak{F}_0| = k\,\delta t \tag{12}$$

This choice places the origin of time based on a universe fragment $\mathfrak{F}_0$ that we can consider as the universe before we start counting time. More importantly, according to eq. (3), the measurement of time is local to $\mathfrak{F}_0$, as required by special relativity: two different observers may measure a different time even if they apply identical processes T and δT to different systems $\mathfrak{F}$ and $\mathfrak{F}'$. We will call T the *measure of time*, δT a *displacement through time*, the corresponding numerical values t is a *time coordinate*, and the numerical value δt is the *time resolution* of our measurement.

The linearity condition in eq. (12) only holds to the extent that T cannot distinguish one δT from another. Often, physical processes are indistinguishable only for a limited number of repetitions: oscillations of a pendulum decay over time if no energy is injected into the system; carbon-14 dating relies on atom decays that are individually indistinguishable, but end up changing the overall atom population measurably over a large period of time. Linearity allows us to consider two measurements of time based on different physical processes as identical (up to a scaling factor), and thus measuring the same time. On the other hand, such a simple scaling-based identity may hold only for short-scale time measurements.

All physical systems evolve through time. Choosing any particular local displacement through time δT is sufficient to define the time evolution of any universe fragment $\mathfrak{F}_0$ using eq. (13).

$$\mathfrak{F}_t = \delta T^k \mathfrak{F}_0 \tag{13}$$

An interpretation of this equation is that the system evolves along time through the same physical process that causes the passage of time we measure with T. The observed evolution of $\mathfrak{F}$ depends on which process δT was picked up. For instance, consider that δT_1 is the beat of a clock you hold in your hand, and δT_2 the beat of an identical clock that stays in the lab. If you start moving, special relativity tells us that the observed evolution of the universe according to δT_1 diverges from the evolution according to δT_2.

Distance Measuring distance is also performed by counting identical physical events in space, for instance how many times a hero's foot would fit between two objects, or how many wavelength of an electromagnetic wave define one meter[7]. Like for time, we can identify a process of displacement through space δX and the associated measure of space X, with a linear definition of a space coordinate x shown in eq. (14).

$$x = |X \delta X^k \mathfrak{F}_0| = k\,|X \delta X\, \mathfrak{F}_0| = k\,\delta x \tag{14}$$

Like for space, the choice of origin for a distance measurement is based on a particular system $\mathfrak{F}_0$ that we consider as the "center of the universe" for the measurement being chosen. The measurement of space is local to $\mathfrak{F}_0$ as required by special relativity. Like for time, X is the measure of distance, δX a displacement through space, x is a spatial coordinate, and δx is the resolution of our distance measurement.

There is an added twist for space because one cannot find a single choice for the displacement of the system through space. Instead, experience shows that we need three locally independent displacements through space to represent the displacement between two arbitrary systems. A generalized local displacement through space is given by an equation like eq. (15).

$$\mathfrak{F}_{xyz} = \delta X^x \, \delta Y^y \, \delta Z^z \, \mathfrak{F}_0 \tag{15}$$

The processes δX, δY and δZ, as well as the associated measurements X, Y and Z, can be chosen to be identical to one another by rotation. In the rest of this article, we will ignore the complication of choosing processes that are not identical through rotation, even if this is often done in practice: one may very well be measuring altitude using air pressure while latitude and longitude are measured relative to the ground.

Space-time displacement Displacement through space and time can also be chosen to be locally independent from one another: space-time is locally Minkowskian. A more general form for the displacement of a system through both space and time is given by eq. (16):

$$\mathfrak{F}_{xyzt} = \delta X^x \, \delta Y^y \, \delta Z^z \, \delta T^t \, \mathfrak{F}_0 \tag{16}$$

Adopting standard terminology, a choice of four independent measurements X, Y, Z and T is a choice of space-time coordinates, since it dictates the corresponding coordinates x, y, z, and t. Following a common general relativity convention, we can index the coordinates and call them X_i, where X_0 is time and X_1, X_2, X_3 are spatial coordinates[5].

As long as the processes δX_i and X_i exists physically, we can generate any particular coordinate value x_i (up to a resolution δx_i) by applying the corresponding displacement enough times, according to eq. (12) and eq. (14). For that reason, the corresponding individual displacements δX_i can be called the *generators* of the coordinates X_i. On the other hand, that does not necessarily imply that we can reach all points of the universe using a particular choice of coordinates.

Change of coordinates The choice of coordinates X_i is not unique. As suggested in section 2.4, we must now consider two different kinds of coordinate changes:

- Coordinate changes where quasi-identical physical processes are used, and
- Coordinate changes due to a change in the physical processes being used.

The theory of general relativity focuses on the first kind of coordinate changes, called a "change of reference frame". Implicitly, general relativity assumes that space and time coordinates as measured in a given reference frame are unique and independent of the physical process by which they are being measured.

3.2 Special Relativity

The definition of spatial coordinates given in the preceding section does not correspond to the traditional definition of space-time as a 4-dimensional continuum generally used in special relativity. We must now find a way to reformulate the original principles of special relativity using only measurements, ideally without making space or time unreasonably pre-eminent.

Continuum vs. Discrete Equations used in special and general relativity are problematic when analyzed from a TIM perspective. Albert Einstein already pointed out that the choice of a Gaussian metric is only possible if small enough domains of the continuum can be considered Euclidean[8]. Therefore, an implicit requirement of the formulation of general relativity is that physical entities, including space-time, energy and momentum, are continuously differentiable. Unfortunately, using the TIM definition, space and time measurements are not even continuous, since they are reported using a discrete graduation, and as we discussed above, generally based on a discrete count of identical physical processes.

5. Since we use an exponent notation to count applications of a process, it is less confusing to use a "lower index" notations when indexing physical processes, even when an "upper index" notation like x^a would typically be used in general relativity.

However, any measurement for which the linear approximation eq. (11) is valid can be identified with a linear function m of the measurement coordinates k_i, and that linear approximation $m(k_1, ..., k_n) = \sum k_i\, m_i$ is continuously differentiable on $\mathbb{R}^n$ with respect to each k_i. This makes it possible to replace the actual measurements with a continuously differentiable function that matches any possible measured value.

This is true in particular of space and time coordinates. We can replace the actual measurement $x(k) = k\,\delta x$, $k \in \mathbb{N}$ with the *continuous extension* $x(k) = k\,\delta x$, $k \in \mathbb{R}$. The latter is continuous and differentiable, and it matches all possible measured values of x. This approximation is perfectly legitimate in practice even for differentiation, because the number of repetitions of the fundamental processes δX_i is usually chosen to be very large (for instance, there is an extremely large number of atoms in the typical tape measure), so that the resolution δx_i is very small relative to x_i. In this "smooth" limit, there is no practical numerical difference between the limit $\frac{\mathrm{d}f}{\mathrm{d}x}(x)$ and an approximation like $\frac{f(x + n\,\delta x) - f(x)}{n\,\delta x}$ when n is sufficiently small.

From physical to mathematical rotation A similar, but more subtle problem occurs when we write relations between the results of various measurement processes, because the simplest form of these relations may involve continuous mathematics. This is the case in particular for the mathematical rotations that we associate with physical rotations.

We observe experimentally that the relations between measurements of distance in space give them the structure of Euclidean geometry[6]. A common formulation of Euclidean geometry uses real numbers and points in $\mathbb{R}^n$ (where $n = 3$ is used to model physical space). There are good reasons for this choice, which are evident if you try to build an Euclidean group over $\mathbb{N}^n$ for example: $\mathbb{N}^n$ is not closed under arbitrary rotations. This may appear to be a show-stopper for the TIM, where measurement results are by construction isomorphic to subsets of $\mathbb{N}^n$, as visible in eq. (11). But in practice, we can retain the Euclidean group on the continuous extensions. This structure will allow us to make predictions at an infinite resolution, and from there, predictions at the actual resolution.

For example, if your only measurement apparatus has a resolution of 1 cm, and if you rotate an object of length 10 cm by 45 degrees, the predicted mathematical length based on the continuous extension above will be $10 \cos(45) = 10\,\frac{\sqrt{2}}{2} \simeq 7.07$. It is however unreasonable to claim that your measurement apparatus will give you the result 7.07 cm (or, worse yet, the exact irrational value), since its resolution is only 1 cm. We may have taken the bad habit of saying "it will measure between 7 and 8 cm" because our eye has a much higher resolution than the graduations on typical measurement tools like pocket rulers. But in reality, the result returned by a physical measurement with resolution 1 cm should be 7 cm or 8 cm depending on the instrument.

Problem solved by special relativity The original presentation of special relativity introduced the theory using concepts such as inertial reference frames. These concepts are slightly complicated to translate in terms of measurements. On the other hand, the constancy of the speed of light is much easier to express. Historically, this was also the fundamental issue that special relativity tried to address, both from an experimental point of view (Michelson-Morley experiment) and from a theoretical one (justifying why speed of light in Maxwell's equations was $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, and therefore independent from the speed of the observer).

As we now know, the solution to this problem revealed that space-time has a Minkowski structure, where the distance is written using an equation like $\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$ and is identical for all observers in a well-chosen classes of observer ("inertial reference frames"). For the same class of observers, the equation expressing the distance should also hold in the TIM, but we may need to specify how the observers measure $\mathrm{d}s$ or $\mathrm{d}x$.

Euclidean space Let us start with the observation that, at least for well-chosen measurements of distance (in the same sense as we talked about a well-chosen class of observers), we can give our measurements of spatial distance on a physical plane the structure of a 2-dimensional Euclidean group. The distance can be written as $\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2$. As explained above, this is really a continuous extension of discrete observations approximately verifying Pythagoras' theorem. The mathematics corresponding to this particular mathematical structure are well known.

6. Like for counting, it is reasonable to suppose that our species initially learned Euclidean geometry from these observations, not from axiomatic reasoning.

For systems that can be assimilated to a point moving in space along a trajectory, we perform the measurements of space-time coordinates relative to the individual systems. In eq. (15), this reference system was denoted as $\mathfrak{F}_0$. To convert the coordinates x, y as measured from system $\mathfrak{F}_0$ to the coordinates x', y' as seen from system $\mathfrak{F}_0'$, we must consider the translation and rotation required to bring one tangent referential system to another.

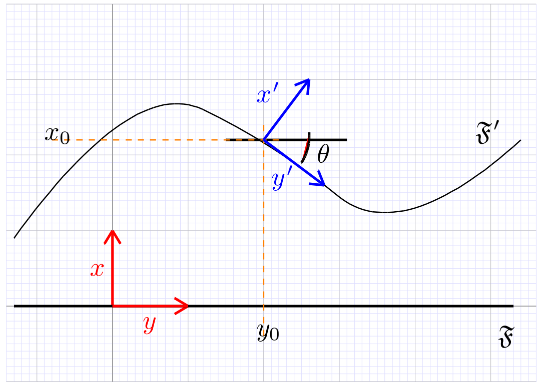

Figure 1. Change of coordinates

The coordinate changes between (x, y) and (x', y') for the two-dimensional example shown in figure 1 can be written as a combination of:

- a translation by a vector (x_0, y_0) where x_0 and y_0 correspond to the measurement relative to $\mathfrak{F}$ of the point that measures as $(0, 0)$ relative to the system $\mathfrak{F}'$.
- a rotation by angle θ.

The general formulation of the coordinates change is well known and given by eq. (17):

$$\begin{cases} x' = (x - x_0)\cos\theta - (y - y_0)\sin\theta \\ y' = (x - x_0)\sin\theta + (y - y_0)\cos\theta \end{cases} \tag{17}$$

The relations written in eq. (17) are exact over real numbers. As discussed earlier, they are always approximate with actual physical measurements, as can be seen on figure 1 if we postulate that the grid indicates the resolution of the measurement relative to system $\mathfrak{F}_0$.

If the coordinates of the trajectory of system $\mathfrak{F}'$ relative to system $\mathfrak{F}$ are parameterized as continuously differentiable functions $x(s)$ and $y(s)$, we can also express cos θ and sin θ as eq. (18):

$$\begin{cases} \cos\theta = \dfrac{\frac{\mathrm{d}y}{\mathrm{d}s}}{\sqrt{(\frac{\mathrm{d}x}{\mathrm{d}s})^2 + (\frac{\mathrm{d}y}{\mathrm{d}s})^2}} \\ \sin\theta = \dfrac{\frac{\mathrm{d}x}{\mathrm{d}s}}{\sqrt{(\frac{\mathrm{d}x}{\mathrm{d}s})^2 + (\frac{\mathrm{d}y}{\mathrm{d}s})^2}} \end{cases} \tag{18}$$

Assuming $\frac{\mathrm{d}y}{\mathrm{d}s} \neq 0$, another formulation is eq. (19):

$$\begin{cases} \cos\theta = \dfrac{1}{\sqrt{1 + (\frac{\mathrm{d}x}{\mathrm{d}y})^2}} \\ \sin\theta = \dfrac{\frac{\mathrm{d}x}{\mathrm{d}y}}{\sqrt{1 + (\frac{\mathrm{d}x}{\mathrm{d}y})^2}} \end{cases} \tag{19}$$

The reasoning above can be made entirely in $\mathbb{R}^n$ ($\mathbb{R}^2$ for the figure above) based on the axioms of Euclidean geometry. It is a purely mathematical reasoning. However, as shown earlier, it also applies to the symbolic results of measurements, i.e. to the physical world, but *only to the extent that Euclidean geometry applies to them as well*. I can choose a calibration for my spatial measurements so that Euclidean geometry relations become inexact, and then none of the equations above can be considered true for such measurements.

Space-time coordinate changes One of the essential findings of special relativity was that the same reasoning holds when considering a coordinate change involving both a space and a time coordinate, with a significant difference: the numerical distance s that is preserved verifies a Minkowski metric $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2$ or $\mathrm{d}s^2 = \mathrm{d}x^2 - c^2\,\mathrm{d}t^2$ instead of the definite positive Gaussian metric $\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2$ that is preserved between space coordinates.

There are interesting consequences to a distance written as $\mathrm{d}s^2 = \mathrm{d}x^2 - c^2\mathrm{d}t^2$. Compared to the Euclidean metric, it can be seen as a transformation $y \rightarrow i\,c\,t$ where $i^2 = -1$, a transformation known as *Wick rotation*. Whereas for classical rotation between two space coordinates, $\cos\theta \leqslant 1$, for rotations between a space and a time coordinate, $\cos\theta \geqslant 1$, since it takes the mathematical form of an hyperbolic cosine. It is easy to verify that what we wrote as $\cos\theta$ takes the form shown in eq. (20), traditionally denoted as γ in special relativity textbooks. Similarly, $\sin\theta$ is traditionally written as $i\,\beta\,\gamma$. In that case, eq. (17) transforms into the well-known Lorentz equation shown in eq. (21).

$$\cos\theta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \tag{20}$$

$$\begin{cases} x' = \gamma(x - x_0) - \gamma\,v\,(t - t_0) \\ t' = \gamma(t - t_0) - \frac{1}{c^2}\gamma\,v(x - x_0) \end{cases} \tag{21}$$

The value of this reasoning is not in the mathematics, which is well known, but in the possibility to do it under the conditions imposed by the TIM. As long as the Minkowski metric condition $\mathrm{d}s^2 = \mathrm{d}x^2 - c^2\mathrm{d}t^2$ holds between the measurements we chose, the Lorentz transformation and, consequently, the known effects of special relativity remain valid.

Special relativity effects The reasoning above did not involve any notion of "inertial reference frame" or "non-accelerated observer". This arguably makes explaining some well-known special relativity effects easier, since they can be described by analogy with what we usually call "perspective" in the case of space-space rotations. Naturally, the transformation $y \rightarrow i\ c\ t$ means that the effects will be slightly different. Perspective induces contractions for space-space rotations that are related to $\cos\theta \leqslant 1$, and so it induces dilatation for space-time rotations where $\cos\,\theta \geqslant 1$. Conversely, dilatation effects for space-space rotations that are related to $\cos\,\theta$ turn into contractions for a space-time rotation.

This simplification is particularly noticeable for special-relativity effects that involve some acceleration. Those are often considered tricky to deal with in the more traditional formulation of special relativity. For instance, the so-called "Langevin's twins paradox"[9] is illustrated on figure 2. The straight arrow along the horizontal axis represents the trajectory of a twin who remains on earth. The curve represents the trajectory of a twin who travels in space at high speed. For space-space rotations, as shown on the figure, the curve is always longer than the straight line. It is easy to verify that the elongation factor takes the form of $\int\ \frac{1}{\cos\,\theta}$ along the curve, which is always greater than 1 when $\cos\,\theta \leqslant 1$. On the contrary, if $\cos\,\theta \geqslant 1$, then the curve is always shorter than the straight line: less time elapsed for the moving twin than for the twin who remained on earth.

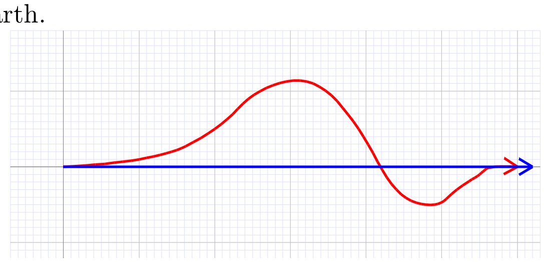

Figure 2. Twins paradox

Space-time and electromagnetism A relation observed between spatial measurements, such as $\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2$, is not overly surprising, since it relates measurements that we can transform into one another by physical rotation. But if such a relation indicates that essentially the same physical process is at play for x and y, shouldn't we also infer the existence of a single physical process behind measures of x and t from the Minkowski metric relation $\mathrm{d}s^2 = \mathrm{d}x^2 - c^2\,\mathrm{d}t^2$?

What process can this be? The presence of c in the metric formula suggests that this should be related to the propagation of electromagnetic waves. Is it reasonable to see electromagnetism as the foundation for the measure of distance and time? At a macroscopic level, this certainly seems plausible. Our definition of the meter is now based on light waves. Measurements involving rods use the properties of solids which are held together primarily by electromagnetic forces. Our definition of time is itself based on the de-excitation of atoms, and this is the fundamental process generating photons.

The traditional formulation of electromagnetism is written using a space-time background, on which electromagnetic fields are written. In this view, space and time exist even if there is no electromagnetic field or interaction. However, the remarks made above suggests that this hypothesis may be incorrect. The properties that we have traditionally attributed to space and time might express relations defined primarily by electromagnetic interactions[7].

3.3 Non-Euclidean Geometry

The reasoning in the previous section was articulated around observed properties suggesting a locally Euclidean or Minkowskian geometry. One of the fundamental innovations of general relativity was to formalize the more general case where the propositions of Euclidean geometry do not necessarily hold. We will now see that the need for this non-Euclidean formulation not only remains in the TIM, but can be justified for new reasons.

Non-commutativity The reasoning in the previous section implicitly requires that displacement through space and time coordinates commute locally. For example, eq. (16) is valid only to the extent that we cannot distinguish through any measurement between the state of the universe defined by $\mathfrak{F}_{xy} = \delta X\,\delta Y\,\mathfrak{F}_0$ and $\mathfrak{F}_{yx} = \delta Y\,\delta X\,\mathfrak{F}_0$. Otherwise, eq. (16) would predict two different evolutions of the universe depending on which path is chosen, and this contradicts our original definition of the notation $\mathfrak{F}_1 = \mathfrak{F}_2$.

The fact that displacements commute locally does not imply that they still commute after being repeated a large number of times[8]. In other words, we cannot deduce $\delta X^n \delta Y^m \mathfrak{F} = \delta Y^m \delta X^n$ from $\delta X^j \delta Y^k \mathfrak{F} = \delta Y^k \delta X^j$ whenever $n > j$ or $m > k$. As a matter of fact, at a large enough scale, we observe that physical displacements through space-time do not generally commute.

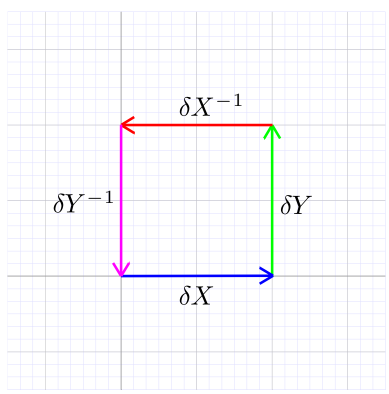

Figure 3. Commuting displacements

7. It would be particularly ironic, if this hypothesis proves to be correct, that Einstein's quest for a theory unifying gravitation and electromagnetism was, in a sense, already fulfilled with his own general relativity.

8. This statement may seem mathematically contradictory, if one forgets that the TIM "equality" is based on what we know and can physically measure. It is experimentally possible for two processes to be indistinguishable when repeated a small number of times, but to diverge at larger scale. One has to remember that the set of physical hypotheses is part of the definition of equality we use, and the maximum number of applications of a physical process can be part of these hypotheses.

To illustrate this, consider mobiles subjected to gravitation forces keeping them at the surface of our planet. In a room, moving one meter forward followed by one meter sideways brings you practically at the same point as moving one meter sideways followed by one meter forward. The displacement in figure 3 is is practically indistinguishable from "returning to the same point". Considering for simplicity that Earth has a radius of roughly 6000km, the angle α seen from the center of Earth for a 1m displacement is at first order $\alpha = \frac{1m}{6000\,10^3 m}$. The curvature error is in the order of 6000km $(1 - \cos\alpha)$, and for such a small value of α we can rewrite that as 6000km $\frac{\alpha^2}{2}$, so the displacement error made by assuming that earth is flat is less than 10^{-7}m. Except under the most stringent laboratory conditions, this error is simply negligible.

However, if the processes "moving one meter forward" and "moving one meter sideways" are repeated 10^7 times, the distance in each direction is 10000 km, or approximately a quarter of the circumference of earth. In that case, the two points you will reach are about 10000 km apart, as illustrated on figure 4 as seen from a pole.

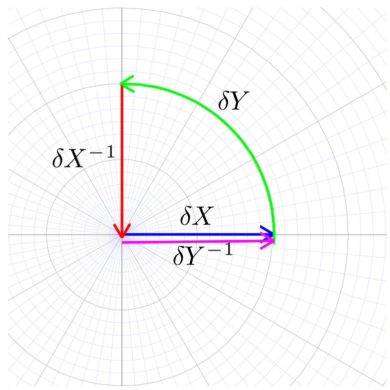

Figure 4. Non-commuting displacements

Qualitatively, the relationship "X and Y commute" is therefore very different between the two cases. However, from the local point of view of the mobile, there is no difference in the local movements between figure 3 and figure 4. In both cases, we are moving for example "N steps forwards, N steps left, N steps backwards, N step right".

In this example, the non-commutativity of space-time displacements appears related to curvature. This is not surprising, as the Riemann curvature tensor measures the non-commutativity of the covariant derivative. Intuitively, by following displacements along two different displacements δX and δY, we are reconstructing in physical terms the parallel transport normally used to evaluate the Riemann curvature tensor.

Geodesics One might be tempted to think that non-commutativity in the example above is simply due to a bad choice of coordinates, because on earth we are not moving "straight" but along the "curved" surface of earth. By this reasoning, the problem would disappear if we followed a "straight line" along each axis. Our space and time measurements would then correspond to an Euclidean geometry.

Unfortunately, in physical terms, it is difficult to define what a "straight line" is. Arguably, one of the best physical definitions of a straight line is the path followed by a ray of light, yet a number of physical experiments show two or more light rays starting at the same time from the same point and reaching the same point at the same time. A simple optical lens shows this property, but even in the vacuum, the property appears at large scale (gravitational lensing[10], Shapiro delay[11]). It also show up in tabletop experiments, for example in the presence of obstacles (Young's slits experiments). The propagation of light is therefore not a very solid foundation to build an Euclidean geometry on.

A similar problem exists for other definitions of straight lines (or geodesics). For instance, one could define geodesics as the paths followed by "free-falling observers". One space measurement would be points separated by a constant local time interval along the path. But in that case too, one can find two geodesics with the same start and end points in space-time, for instance two opposite trajectories along the same orbit.

Non-Euclidean geometry Mathematics tell us that if there are more than one straight lines between two individual points, then the geometry is not Euclidean. Since the best definitions of a straight line we have in the physical world exhibit this non-Euclidean property, we need to give up the possibility to associate an Euclidean geometry to any system of coordinates based on physical space and time measurements.

The justification given here is different from the reason given by Albert Einstein to postulate a non-Euclidean geometry for space-time. His original motivation appears to have been an "esthetic" desire to write a formulation of physics that did not favor any particular coordinate system. But it was not, at the time, required to match strong experimental evidence, and it certainly introduced a lot of mathematical complication. The justification for this complication really came *a posteriori*, when predictions of the theory were validated by experiments.

In an introductory book, Einstein also cited as a justification the existence of non-Euclidean space-time hyper-surfaces predicted by special relativity, such as the surface of a rotating disk along its proper time[12]. But that observation alone is not sufficient to prove that space-time itself is curved, just like the existence of spheres in space does not prove that space is curved. Indeed, the hyper-surface selected for Einstein's example was defined by the path in space-time of the points along the radius of a rotating galaxy. This hyper-surface is analogous in space-time to an helicoid in space, and an helicoid is not an Euclidean surface. It is unclear how much thought Einstein really gave to this particular example, one among many in the book.

The situation is different today. With the experimental evidence that we now have, we need non-Euclidean geometry in the TIM because we do not know how to define an Euclidean geometry relating space-time measurements, not because of any "first principle" consideration.

3.4 General Relativity equations

A reminder of the traditional formulation of general relativity[5] is useful to see how the TIM may impact it. In the following equations, we will use Einstein's convention of summing terms with identical lower and upper indices. We will therefore avoid references to repeated processes using the notation P^n to avoid confusion.

Mathematical tools Many of the formulas in general relativity are mathematical tools that do not need any matching physical reality to be used legitimately. There is no issue using them to manipulate the extension to $\mathbb{R}^n$ of any linear approximation function $m(x_1, ..., x_n)$, including, but not limited to, the space-time coordinates themselves. These mathematical tools can, in particular, be used freely on mathematical spaces that have a different number of dimensions than the 4 dimensions we observe in the physical space-time.

Let's consider a set of coordinates x_i in $\mathbb{R}^n$. A Gaussian metric g is a rank (0,2) symmetric tensor defining a measure of distance from individual coordinates, as shown in eq. (22). The inverse of $g_{\mu\nu}$ is written $g^{\mu\nu}$.

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \tag{22}$$

The Levi-Civita connection ∇ defines a derivative D for a vector field V along a curve γ as $D_t V = \nabla_{\gamma(t)} V$. The covariant derivative generalizes partial derivative, adding a term that is the change caused by curvature itself, so it writes as $D_\mu A^\nu = \partial A^\nu + \Gamma^\nu_{\rho\mu} A^\rho$. The added term uses the Christoffel symbol $\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda}(\partial_\nu g_{\lambda\mu} + \partial_\mu g_{\lambda\nu} - \partial_\lambda g_{\mu\nu})$.

The covariant derivatives do not commute, and the Riemann curvature tensor $R^\lambda_{\rho\mu\nu}$ is a rank (1,3) tensor defined as $R(u, v, w) = \nabla_u \nabla_v w - \nabla_v \nabla_u w - \nabla_{[u,v]} w$. For the coordinate vector fields $u = \frac{\partial}{\partial x_i}$ and $v = \frac{\partial}{\partial x_j}$, $[u, v] = 0$ and the Riemann tensor measures the commutator of the covariant derivative, i.e. $R^\lambda_{\rho\nu\mu} A_\lambda = (D_\mu D_\nu - D_\nu D_\mu) A_\rho$.

The Ricci tensor is defined from the Riemann tensor in coordinate terms as $R_{\mu\nu} = R^\rho_{\mu\rho\nu}$. We can then define the Ricci scalar by raising an index to get $R^\mu_\nu = g^{\mu\nu} R_{\mu\nu}$ and then contracting to obtain the Ricci scalar $R = R^\mu_\mu$.

None of these equations makes any hypothesis about the physical universe. They play, in a more complicated form, the same role as mathematical differentiation, rotations or translations in our discussion of special relativity. For the same reason as was discussed in the special relativity section, we can therefore accept them as purely mathematical relationships in the TIM. More precisely, all these relations can be considered as holding as long as we can write the metric for 4-coordinates as shown in eq. (22), just like the coordinate transformations in special relativity held as long as the Minkowski metric did[9].

Physical values In contrast with mathematical tools, if we focus on gravitation, ignoring other forces, there are two general relativity quantities that relate directly to physical measurements. The space-time metric $g_{\mu\nu}$ relates to space and time measurements. The stress-energy tensor $T_{\mu\nu}$ relates to mass, energy or momentum measurements. These two entities definitely require a TIM analysis.

Another quantity appears in the equations, at least for some formulations. The cosmological constant Λ relates to the shape of the universe at large scale. The status of the cosmological constant is slightly more difficult, since it does not represent an intuitive measurement from our everyday life. Measuring it is much more difficult[13], and mechanisms are still being proposed to explain its emergence and evolution over time[14]. Arguments have even been presented that a cosmological constant makes our own existence improbable[15]. This makes a non-speculative discussion of this particular quantity significantly more difficult.

Metric and experimental choice The Gaussian metric $g_{\mu\nu}$ as defined above relates a choice of distance $\mathrm{d}s^2$ to small variations $\mathrm{d}x_\mu$ of the individual space or time measurements x_μ, which result of a particular choice of coordinate measurements X_i. Provided the measurements give definite, non probabilistic results, there is no particular problem computing a specific metric from space and time measurements, i.e. the factors $g_{\mu\nu}$ from the individual x_μ.

There is, however, a problem in choosing which physical processes are equivalent to a space-time metric in general relativity. Clearly, we know intuitively that mass or electric charge measurements are in general not ideal methods to measure a spatial or time coordinate. But it is difficult to explain exactly why. Given an arbitrary measurement system, how do you recognize that it is measuring a distance or a metric?

Furthermore, when choosing between different measurements that we would intuitively consider as space or time measurements, picking the best one is not always immediately obvious. Imagine that we must choose between measurements made with a metal rod and measurements made by measuring the trip time of a laser beam. What if the metal rod becomes really hot, causing it to change dimensions or even melt? What if the laser beam has to travel through water, where the high refractive index with alter both the path and trip time of the light? External factors influence the physical measurements, but not in the same way.

In the context of the TIM, we need to identify precisely which specific physical measurements the theory of general relativity actually talks about. Following the remarks made about special relativity, and based on the kind of verifications already performed for general relativity, it is reasonable to postulate that the only measurements for which the field equation of general relativity is valid use metric measurements that are practically equivalent to the propagation of electromagnetic waves in a vacuum. Other measurements can be considered equivalent to the extent that they are primarily defined by such propagation. For instance, solid rods are held together primarily by electromagnetic interactions between atoms, and we know that solid matter is made primarily of empty space.

Stress-energy tensor The stress-energy tensor T is a rank (0,2) tensor that indicates how energy and momentum flow. Specifically, $T(u,\, v)$ is the flow along direction u of momentum along direction v, where energy is represented by the momentum along the time dimension, and the flow along the direction of time represents a density.

9. Obviously, more physical measurements will verify a general Gaussian metric than a Minkowski metric.

Like for the metric, we have a problem defining precisely what measurement is valid to measure T. At first sight, a measurement of mass at rest with respect to the measurement apparatus seems relatively easy enough to define: we put the mass on a scale. But this simple definition relies on the reaction of the mass to the pull of gravitation.

A more general choice is to define it as a linearization of the property of individual particles using eq. (11), where the m_i are chosen to balance out the reaction to an arbitrary force of a body made only of particles i and of a body made only of particles j. In other words, we define system α as being twice as massive as system β because system α reacts to an external force like two systems β put together.

But this definition remains problematic, because it does itself rely on acceleration, and acceleration itself is defined based on space and time measurements. It would be more satisfying if we could avoid such an implicitly recursive definition of the mass.

This may not be entirely possible. For a single particle whose position is being measured using the "radar method", the Schwarzschild metric can be interpreted as the appearance of a mass-dependent proper time delay $\chi = 4\,G\,m/c^3$ between the absorption and the re-emission of a photon by the particle[16]. While it remains unclear how this mechanism would extend to multiple particles, it seems to suggest that mass can be identified to a proper time delay in some fundamental interaction, and therefore cannot necessarily be defined in a way that does not depend on measurements of time.

Cosmological constant The cosmological constant Λ appears in the most general formulation of the Einstein tensor, as defined by eq. (23).

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\; g_{\mu\nu} - \Lambda g_{\mu\nu} \tag{23}$$

The value of the cosmological constant can only be deduced from the observations at large scale of the universe. There is no strong reason at this point to believe that all choices of coordinates X_i would result in the same observed cosmological constant. We can speculate that Λ is a parameter that depends on the choice of physical measurement defining the coordinates. It may for example be an emergent property derived from time evolution of the stress-energy tensor[14].

Field equation The law of gravitation in general relativity relates the stress-energy tensor T to the Einstein tensor G as in eq. (24), where we choose $\kappa = \frac{8\,\pi\,k_G}{c^2}$ to match observations, k_G being Newton's gravitational constant.

$$G = \kappa T \tag{24}$$

As noted earlier, we have some choice for the measurements we use for space/time coordinates, and even if they agree to a very good extent locally, they may not agree at a large scale. If we have two possible choices for X that lead to different values for G at very large scales, the value $\kappa\,T$ cannot possibly be equal to both.

We may have already observed this effect. The problem of the "dark matter"[17] or the anomalous acceleration of the Pioneer spacecraft[18] may be some of the first indications of a divergence at large scale between the measurements we chose for space and time compared to the measurements we chose for energy or mass.

Observations of cosmic events, such as the galactic collision in the "Bullet cluster", have been widely publicized as demonstrating the existence of dark matter[19]. However, they are based on very indirect reconstruction of the gravitational field, by means of statistical analysis (weak gravitational lensing). Our own atmosphere shows a significant tendency to create artifacts in the propagation of light. We must keep in mind the possibility that gases or dust that result from galactic collision would also cause various distortions of light. In any case, dark matter at this point is not a settled topic[20].

Existence of a Minkowskian metric In general relativity, the equivalence principle of gravitation and inertial acceleration is expressed as the existence of coordinate systems where the metric is locally Minkowskian, and where the equation of motion of a free particle moving along 4-vector u_μ is $D\,u_\mu = 0$.

For the present theory, we postulated instead that:

1. we can find for any system $\mathfrak{F}_0$ four repeatable time and space displacements processes δX_i that are mutually independent.

2. the evolution of the system $\mathfrak{F}_0$ over space and time follows the local time displacement $\delta T = \delta X_0$ corresponding to the "passage of time", i.e. $\mathfrak{F} = \delta T^k \mathfrak{F}_0$.

This implies the existence of a local Minkowski metric. Since the δX_i are repeatable and indistinguishable by measurement X, the corresponding measurements x_i obey the linear approximation. It is possible to associate a metric $g_{\mu\nu}$ to the x_i. The hypothesis that the δX_i are independent implies that the metric is diagonal. By scaling of the various x_i, we can select a metric which has only $+1$, -1 on the diagonal. No scaling is even necessary if we used identical processes for the various X_i and δX_i.

We can't have zeroes on the diagonal, as this would correspond to a non-observed physical situation where a displacement along one axis does not change the distance measurement. This is particularly true for any measurement based on counting electromagnetic wavefronts: there is no way to select a space or time coordinate along which light is standing still.

The second postulate then requires that one of the four coordinates (corresponding to time) be different from the others. We end up with a local metric verifying $\delta s^2 = \delta x_0^2 - \delta x_1^2 - \delta x_2^2 - \delta x_3^2$ or $\delta s^2 = -\delta x_0^2 + \delta x_1^2 + \delta x_2^2 + \delta x_3^2$, i.e. a Minkowski metric.

Geodesics The postulate that the passage of time is a physical process δT indicates that, locally, the measurement T will result in linearly increasing time values t. The condition that the δX_i are mutually independent implies that at short scale, there is no linear dependency on time of the chosen local Minkowskian space coordinates x_i for $i = 1, 2, 3$. This yields the local geodesic equation $D u_\mu = 0$.

The reasoning leading from the local formulation to the formulation of the geodesic equation in arbitrary coordinates, $\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\nu\rho} \frac{\mathrm{d}x^\nu}{\mathrm{d}s} \frac{\mathrm{d}x^\rho}{\mathrm{d}s} = 0$, is purely mathematical, and consequently applies in the context of the TIM as well.

Electromagnetism and "forces" For gravitation, general relativity does away with the notion of "force", since the movement of a body follows a geodesic. This is not a general case, however, since the same treatment does not apply to other known forces.

For instance, the treatment of electromagnetism in general relativity introduces additional quantities, specifically, the electromagnetic field tensor $F^{\mu\nu}$ and the 4-vector electromagnetic current J^ν. These two tensors are related by the equation $D_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$. The current J^ν is a conserved quantity, so it verifies $\partial^\lambda F^{\mu\nu} + \partial^\mu F^{\nu\lambda} + \partial^\nu F^{\lambda\mu} = 0$. The effect on a charged object is then given by the impact on the momentum-energy 4-vector P^μ of the electromagnetic field $F^{\mu\nu}$, given by the field equation eq. (25):

$$D_t P^\mu = \frac{q}{m} P_\nu F^{\mu\nu} \tag{25}$$

These laws of electromagnetism in general relativity are not obviously related to space-time curvature. In particular, eq. (25) is nothing less than the introduction of a "force" in general relativity. General relativity does not do away with forces, even if it is generally seen as giving an interpretation of gravitation that is based on purely geometric considerations[21, 22]. The movement predicted by eq. (25) cannot be reduced to any curvature of space-time, because it depends on the charge of the particle. A single space-time curvature cannot explain why one particle would turn left while another with exactly the same properties but the charge would turn right. Today, Einstein's many attempts at giving a geometric formulation of multiple forces using for example torsion[23, 24, 25] are generally regarded as unsuccessful.

General relativity therefore did not dispense with the notion of "force" entirely. While it showed that specific geometric properties are equivalent to a force, all forces are not necessarily of geometric nature or origin. This seems at odds with Einstein's formulation of first principles like the principle of general relativity. But this is exactly the conclusion one would expect based on the measurement relativity principle, because geometry gives a preferred role to space and time coordinates, which the measurement relativity principle precludes.

3.5 Scale Relativity

General relativity is really constructed on a number of symmetries which are taken as axioms. One of these axioms, which we already discussed, is continuous differentiability of space and time coordinates. In that model, you can zoom on a system as much as you want, and still expect space-time to look similar. This is not a property we observe with measurements, however. We know that we can cut a pocket ruler in two halves, and then again, and then again. But we cannot do that beyond some microscopic scale, because the laws of physics change. You need a minimum number of atoms for your ruler to be a fixed-size solid.

Zoom As we change scale, or *zoom*, we may need to change the choice of space and time measurement for practical reasons. We therefore replace one choice of coordinates $X_{i/\lambda}$ valid at scale λ with another choice of coordinates $X_{i/\mu}$ valid at scale μ. In order for the two choices of coordinate $X_{i/\lambda}$ and $X_{i/\mu}$ to measure the same physical locations, there must be a large range of systems $\mathfrak{F}$ where the two measurements give the same result, i.e. they verify eq. (26):

$$|X_{i/\lambda}\mathfrak{F}| = |X_{i/\mu}\mathfrak{F}| \tag{26}$$

Using eq. (14), we obtain eq. (27) which indicates that the two measurements are related by a linear relation.

$$x_i = k_{i/\lambda}\delta x_{i/\lambda} = k_{i/\mu}\delta x_{i/\mu} \tag{27}$$

As discussed earlier, these identities are really based on what we know. They do not imply an identity of the physical processes themselves. There may be cases where $X_{i/\lambda}$ and $X_{i/\mu}$ do not return the same result. The set of systems where eq. (26) is verified is called the *overlapping range* between $X_{i/\lambda}$ and $X_{i/\mu}$.

In particular, eq. (27) does not necessarily hold at all scale, i.e. for all possible values of $k_{i/\lambda}$ and $k_{i/\mu}$. In other words, just because two measurements are equal on a given range of scales does not imply that they are equal at any scale. One may remember the example of the movement of a mobile on the surface of a planet, where the tangent coordinates as measured along a laser beam match the planet coordinates locally, but not at large enough scale.

When zooming on the system and changing the coordinates measurements from $X_{i/\lambda}$ to $X_{i/\mu}$, we can define the zooming factor $\zeta_{\lambda\mu}$ relating actual measurements done with the choice of coordinates $X_{i/\lambda}$ to the actual measurements done with the choice of coordinates $X_{i/\mu}$. Since the measurements themselves are performed by counting physical processes, we define $\zeta_{\lambda\mu} = \frac{k_{i/\mu}}{k_{i/\lambda}}$. Whenever eq. (27) holds, we also have $\zeta_{\lambda\mu} = \frac{\delta x_{i/\lambda}}{\delta x_{i/\mu}}$, which justifies the name "zooming factor": the zooming factor increases as resolution becomes finer.

Renormalization The same laws of physics apply on the overlapping range where eq. (26) holds, whether they are defined using the measurement $X_{i/\lambda}$ or $X_{i/\mu}$. Combined with the linearity of eq. (27), the equality on a large enough range of systems $\mathfrak{F}$ often implies that the laws themselves must remain identical under a change of measurements. This is certainly true for "simple enough" laws, such as laws relating polynomial functions of the coordinates, since it is sufficient in that case to supply a finite number of points to completely define the law.

On the other hand, distant laws of physics, for example laws based on values measured at "infinity", are not guaranteed to remain identical under a change of space-time measurements. The measurements may match locally, yet diverge at large enough scale.

These remarks can be seen as a physical justification for *renormalization*, a common technique when searching for quantized versions of field theories[26]. Renormalization assumes that the laws of physics remain identical when zooming out, after one has eliminated degrees of freedom that are no longer relevant at the scale being considered. When renormalizing, integrals to infinity or around poles often diverge and must be replaced with actual measured values, whereas the local laws of physics remain identical. The reason we need to perform this replacement is really that we gave more credit to a mathematical formula than we should have.

There is an important practical consequence of the reasoning above. While we can consider local laws as valid at any scale, irrespective of the physical process being used, distant laws may depend at small or large scale on which physical measurement was used.

Scale invariance Change of coordinate measurements corresponding to a zoom introduces a new coordinate, the resolution λ at which the measurement is being done. This coordinate is visible in the notation $X_{i/\lambda}$. The resolution may not be sufficient to uniquely define the physical process we are referring to, but it is necessary.

The consequences on the structure of space-time of an explicit resolution coordinates were studied in details by Laurent Nottale. In particular, he postulated the existence of a universal, absolute and impassable scale of nature that is invariant under change of scale[27]. This scale plays for zooming a role similar to c for velocity, and similarly requires a non-Galilean reformulation of the law describing the combination of two changes of scale. This leads to an interpretation of space-time as having fractal properties[28]. A number of interesting results and predictions were made based on this theory, including predictions on the location of planets in solar systems, or finer computations of the coupling constants.

Existence of a constant scale As discussed above, local laws of physics must be invariant through renormalization after zoom (or change of resolution). A constant with the dimension of a scale, Planck's length $l_p = \sqrt{\frac{\hbar G}{c^3}}$, can be extracted from such local laws, most notably Schroedinger's equation (a local law where $\hbar$ appears).

When we discussed special relativity, we observed that the appearance of a constant c in Maxwell's equations was problematic, because this constant, despite having the dimension of a speed, did not transform like other speeds. This made it necessary to introduce a new form of symmetry, the Lorentz transformation, that would preserve c.

We face the same problem with l_p: this constant has the dimension of a length, yet it does not appear to transform like other scales. This seems to suggest that unlike what is generally stated[29], there is indeed a preferred length scale in the universe. If this is true, then Nottale's hypothesis appears necessary, and validates the reasoning leading to a "non-Galilean" law for combining scales.

4 Quantum Mechanics

The TIM formalism also gives us insight into the traditional formulation of quantum mechanics. Actually, the impact on quantum mechanics is probably much more profound than the impact on general relativity. We will demonstrate that we can derive many of the "axioms" of quantum mechanics from our original postulates, including the mathematical formulation of the wave function ψ. This leads to a novel interpretation, which is arguably simpler than the traditional interpretations known as "Copenhagen" or "Many Worlds". More importantly, it suggests a discrete (as opposed to continuous) normalization condition for ψ.

4.1 Predictions and Probabilities

In general, measurements performed earlier on a system $\mathfrak{F}$ are not sufficient to make a detailed prediction of the result of a new measurement. Instead, what we know about $\mathfrak{F}$ allows us to make a probabilistic prediction on the outcome of the measurement.

N-valued measurement The simplest most-general form of measurement symbolic result is a set with n different values $m_1, ..., m_n$. From a physical point of view, we do not need to consider continuous measurements, because they are physically unrealistic, in the sense that the graduation itself cannot be continuous.

For such a measurement M, a *prediction* on the measurement result m is represented by a vector of probabilities (u_i) where u_i is the probability to measure m_i. Such a vector can therefore represent the state of any system $\mathfrak{F}$ as far as the measurement M is concerned.

Ket representation Since u_i represents probabilities, two conditions must be met: $0 \leqslant u_i \leqslant 1$ and $\sum u_i = 1$. These conditions are more easily expressed if we write each probability as a square, i.e. we write $u_i = \psi_i^2$. The condition $\sum \psi_i^2 = 1$ is necessary and sufficient to guarantee both conditions on the u_i, so the ψ_i can be seen as forming a unit vector in $\mathbb{R}^n$, i.e. they belong the the unit sphere $\mathbb{S}^{n-1}$.

Using Dirac's bra-ket notation, we can write the vector as $|\psi\rangle$ and the probability condition as $\langle\psi|\psi\rangle = 1$. One particular vector written as $|k\rangle$ correspond to a certainty that the measurement will return the symbolic value m_k. Its component representation verifies $\psi_i^2 = \delta_{ik}$. Based on the definition of the ψ_i^2, the probability to find the result $|k\rangle$ when the system is in a state $|\psi\rangle$ is $\langle k|\psi\rangle^2$. The various $|k\rangle$ form a basis of $\mathbb{R}^n$.

Physical processes If $\mathfrak{F}_0$ is an initial system state represented by a probability vector $|\psi_0\rangle$, P an arbitrary physical process, $\mathfrak{F} = P\,\mathfrak{F}_0$ the final system state after applying the physical process P, and $|\psi\rangle$ the probability vector in the final system state, then the effect of P as far as we can tell based on measurement M is to transform the probability distribution as $|\psi\rangle = \hat{P}\,|\psi_0\rangle$, where $\hat{P}$ is some arbitrary operator in $\mathbb{S}^{n-1}$ that transforms the probabilities in a way that matches what we know of the effect of the physical process P.

There is no reason for the operator associated to an arbitrary physical process to be linear. For instance, one might consider a physical process ensuring that the next measurement result for M will be m_k. Such a process would transform any probability state $|\psi\rangle$ into a state $|k\rangle$. In general, $\hat{P}$ is not even an injective function. For a measurement M, the operator $\hat{M}$ maps a probability state $|\psi\rangle$ to a random state $|k\rangle$ with probability $\langle k|\psi\rangle^2$. Therefore, this operator gives different (random) results for the same input.

Indistinguishable processes If P_1 and P_2 are indistinguishable processes as far as M is concerned, then the probability outcome should not depend on whether the process P_1 or P_2 is applied to the system $\mathfrak{F}$. Assuming that the system $\mathfrak{F}$ can be constructed with an arbitrary probability state $|\psi\rangle$, we get $\forall\psi$, $\widehat{P_1}|\psi\rangle = \widehat{P_2}|\psi\rangle$, which means that $\widehat{P_1} = \widehat{P_2}$. Processes that are indistinguishable with respect to M are associated with the same operator on $\mathbb{S}^{n-1}$.

Measurements and Eigenvalues If the physical process P is the measurement M itself, the probability vector $\hat{M}\,|\psi\rangle$ indicates a certainty for the measured value m_k and an impossibility for the others, so $\hat{M}\,|\psi\rangle = |k\rangle$. The repeatability condition $|M\,M\,\mathfrak{F}| = |M\,\mathfrak{F}|$ implies that once the measurement result m_k is obtained, the measurement will continue to yield the result m_k. This implies $\hat{M}\,\hat{M}\,|\psi\rangle = \hat{M}\,|\psi\rangle$, or $|k\rangle = \hat{M}\,|k\rangle$. Therefore, the probability state immediately after a measurement is an eigenvector of the operator $\hat{M}$ associated to the measurement process M.

Linearized operator The operator $\hat{M}$ is a possibly non-linear operator over $\mathbb{S}^{n-1}$ verifying $\hat{M}\,|k\rangle = |k\rangle$. If the m_i are real numbers, one can define a linear operator $\check{M}$ defined as taking the $|k\rangle$ as eigenvectors with value m_k. Since the $|k\rangle$ form a basis, $\check{M}$ is defined entirely by the equations $\check{M}\,|k\rangle = m_k|k\rangle$. This operator has two interesting properties: it is linear, and it can be used to compute the expected value (in a probability sense) $E(m)$ of the measurement M for a system described by the probability vector $|\psi\rangle$.

The reasoning is exactly the same as in "traditional" quantum mechanics. Projecting $|\psi\rangle$ over the individual basis vector $|i\rangle$, $|\psi\rangle = \sum \langle\psi|i\rangle\ |i\rangle$. Applying $\check{M}$, we have $\check{M}\,|\psi\rangle = \sum \langle\psi|i\rangle\ \check{M}\,|i\rangle$. Applying the eigenvector equation, we get $\check{M}\,|\psi\rangle = \sum \langle\psi|i\rangle\ m_i|i\rangle$. Taking the inner product by $|\psi\rangle$, we obtain $\langle\psi|\check{M}\,|\psi\rangle = \sum m_i\langle\psi|i\rangle^2$. Since $\langle\psi|i\rangle^2 = \psi_i^2$ is the probability to obtain measurement result m_i, we finally obtain eq. (28):

$$E(m) = \langle\psi|\check{M}\,|\psi\rangle \tag{28}$$

Note however that $\check{M}\,|\psi\rangle$ is no longer on the unit sphere $\mathbb{S}^{n-1}$ in general, so it is no longer a probability vector. This is a practical drawback of $\check{M}$ compared to $\hat{M}$, in particular if 0 is an acceptable measurement value, because we need to renormalize probabilities after applying it.

Linear approximation A measurement M verifying the linear approximation in eq. (11) defines a countable set of possible measurement results $m_{k_1,\ldots,k_n}$, since all the k_i belong to a countable set. We can write the probability vector indicating a certainty that the measurement result is $m_{k_1,\ldots,k_n}$ as $|k_1, \ldots, k_n\rangle$.

By definition of the k_i in relationship to the physical processes P_i in eq. (11), k_i is a count of how many times the process P_i was applied. Thus, operator $\hat{P_i}$ must transform the vector $|k_1, ..., k_i, ..., k_n\rangle$ into the vector $|k_1, ..., k_i + 1, ..., k_n\rangle$. Since the probability vectors form a basis of the probability state, we can define a linear operator $\hat{P_i}$ defined by this relationship, and that operator will generate the same predictions with respect to measurement M. Using the reasoning about identical processes made above, the operator $\hat{P_i}$ in that case can reasonably be assimilated to the linear operator defined by eq. (29):

$$\hat{P_i}|k_1, ..., k_i, ..., k_n\rangle = |k_1, ..., k_i + 1, ..., k_n\rangle \tag{29}$$

Note that this is the shape of a *creation operator* in quantum field theories.

Combined system If we want to evaluate two distinct measurement processes M_1 giving n_1 different results and M_2 giving n_2 different results, the system will be described by two probability vectors $|\psi_1\rangle$ and $|\psi_2\rangle$, and so the state of the system is described in the tensor product $\mathbb{R}^{n_1} \otimes \mathbb{R}^{n_2}$ on the unit sphere $\mathbb{S}^{n_1+n_2-1}$.

If the two measurements are independent, then they have no effect on one another's probability state, and therefore the corresponding operators $\check{M}$ and $\hat{M}$ on the tensor product space are built as tensor products of the original operator by an identity matrix. Note that eq. (3) implies that the effect of the rest of the universe can be ignored.

However, not all physical processes are independent from M_1 and M_2. Using exactly the same reasoning as in traditional quantum mechanics, such physical processes may create "entangled" states in the tensor space.

4.2 Formal Analogy

The structure of the observations made in the previous section is extremely similar to the way traditional quantum mechanics is usually written, provided we replace "*wave-function*" with "*probability vector*", and "*observable*" with "*measurement operator*".

Many formulation of quantum mechanics exist. We will use the following principles[2, 3] to illustrate the parallel with the formalism presented here:

1. The state of a physical system is defined by a ket $|\psi\rangle$ in a Hilbert space $\mathcal{E}$
2. Every measurable physical value $\mathcal{A}$ is described by a linear operator A on $\mathcal{E}$; this operator is an observable, i.e. a self-adjoint operator on $\mathcal{E}$.
3. A measurement of $\mathcal{A}$ can only give as a result one of the eigenvalues of A. In particular, the result is a real number. If A has a discrete spectrum, then the results of the measurement are quantized.
4. The expected value of the observable A for a system in a state represented by $|\psi\rangle$ is $\langle\psi|A|\psi\rangle$.
5. If a measurement on a system in state $|\psi\rangle$ for observable A gives result a, then the state of the system immediately after the measurement is the normed projection of $|\psi\rangle$ on the eigenspace associated to a.
6. The Hilbert space for a composite system is the tensor product of the state spaces for the component systems.

Compare this to some conclusions of the formalism presented here:

1. What can be predicted about a measurement M is represented by a probability vector $|\psi\rangle$ on $\mathbb{S}^{n-1}$ (i.e. a vector of $\mathbb{R}^n$ verifying $\langle\psi|\psi\rangle = 1$), where n is the number of distinct symbolic or numerical measurement results m_k.
2. Every physical process P (including measurements) is associated to a possibly non-linear operator $\hat{P}$ on $\mathbb{S}^{n-1}$ transforming the probability vector $|\psi\rangle$. A measurement M where the m_k are real is associated to a linear operator $\check{M}$ that has the m_k as eigenvalues.
3. When the system is in state $|\psi\rangle$, the probability that the measurement will give the symbolic result m_k is $\langle k|\psi\rangle^2$, where $|k\rangle$ is the vector of $\mathbb{S}^{n-1}$ defined by the components δ_{ik}. $|k\rangle$ verifies the equations $\hat{M}|k\rangle = |k\rangle$ and, if the m_k are real, $\check{M}|k\rangle = m_k|k\rangle$.

4. If the m_k are real, the expected value of the measurement results for M when the system is described by the probability vector $|\psi\rangle$ is $\langle\psi|\check{M}|\psi\rangle$.
5. If a measurement for a system gives result m_k with certainty, then the state of the system immediately after the measurement is described by probability vector $|k\rangle$.
6. The probability state associated to a composite system is the tensor product of the state spaces for the component systems.

This parallel suggests that quantum mechanics can be interpreted as a theory of measurements. In that interpretation, some of the axioms of quantum mechanics, like linearity, appear related to specific experimental conditions or choices of graduation, while others are more general and derived from what physical processes we chose to accept as measurements.

Trajectory measurement The trajectory of a particle can be defined as a set of identical measurements M_{exist} applied to a family of systems $\mathfrak{F}_{xyzt}$, where each measurement determines whether the particle was found or not at coordinates $(x, y, z, t) = (x_i)$. Therefore, the measurement can return two symbolic results `found` when the particle was found to be present in $\mathfrak{F}_{xyzt}$, and `not-found` when the particle was not found to be present.

Since the measurements M_{exist} are two-valued, what can be predicted about the measurement results can be represented by a vector on the unit circle $\mathbb{S}^1$, or, by isomorphism, by a complex number of the form $e^{i\theta}$, where $\text{Re}(e^{i\theta})^2 = \cos^2\theta$ is, for instance, the probability that the particle is found. Therefore, it makes sense to define $\psi(x_i) = e^{i\theta(x_i)}$ as the representation of the probability state that the particle is found for that family of universe fragments.

Normalization of the wave-function A second condition is necessary, however. The various $\psi(x_i)$ are not independent from one another. If we perform M_{exist} at multiple points of space simultaneously in a given frame of reference, only one of them may give a result `found`, all the others must give the result `not-found`. This means that the sum of the probability vectors corresponding to `found` over all the space where we test existence must be at most 1, and exactly 1 if the measured entity cannot escape the measurement apparatus.

We can define a function Ψ with the same phase as ψ but normalized: $\Psi(x_i) = \frac{\psi(x_i)}{\sqrt{\sum_{x_i}\text{Re}(\psi(x_i))^2}}$. The normalization of Ψ guarantees the standard normalization condition for the wave-function in quantum mechanics, i.e. $\langle\Psi|\Psi\rangle = 1$. Preserving the same phase as $\psi(x_i)$ keeps $\Psi(x_i)$ and $\psi(x_i)$ aligned in the complex plane. Therefore, it ensures that any operator $\check{M}$ for which $\psi(x_i)$ is an eigenvector also has $\Psi(x_i)$ as an eigenvector, with the same eigenvalue. Consequently, this also preserves the expected value of $\check{M}$ at point (x_i).

However, in the reasoning above, Ψ would not be normalized by summing over all of space-time, but only over the points of space-time where the existence measurement is being carried out. This remark eliminate the largest portion of the problems that arise when renormalizing the wave-function in quantum mechanics, notably in relationship with the kind of non-Euclidean changes of coordinates required by general relativity. Here, we compute the norm of the wave-function Ψ only on the points of space time where a test is being carried out, and at the time where it could be carried out.

For example, if we place a very large photographic plate, so that it takes one second for light to go from the center to one of the border, the normalization condition for a photon has to be understood as covering points in space-time that are not simultaneous, since the detection, if it happens at the border of the plate, will happen one second later than if it happens in the center. The "wave-function" Ψ does not collapse "instantaneously" as in quantum mechanics, it instead collapses on a light cone centered on the "last known position" of the photon, i.e. the last universe fragment $\mathfrak{F}$ where M_{exist} returned `found`. Seen from a point where the measurement actually finds `found`, the collapse therefore may contain points that are both in the future and in the past. The points that are farther away from the last known position will collapse in the future, while the points that are closer from the last known position have already collapsed.

Planar wave-function in "empty space" In order to validate the analogy of our function Ψ with a wave-function, we need to check that it evolves in a similar way. For simplicity, we will only consider a simple argument regarding its evolution in a vacuum.

If we consider an evolution along individual local-time displacements δT of a particle, if the δT are indistinguishable with respect to M_{exist}, then any two $\widehat{\delta T}$ should be identical. As we noted earlier, in the general case, $\hat{P}$ is not necessarily an injective function, let alone linear. However, in classical theories of physics, measurements are approximated by locally differentiable functions of time, which implies that they are locally linear to the first order (with the form $\frac{\mathrm{d}m}{\mathrm{d}t}\,\delta t$). Applying the reasoning leading to eq. (29), $\widehat{\delta T}$ itself can therefore be assimilated to a linear operator on the probability space. In turn, if $\widehat{\delta T}$ is linear with respect to these various measurements, it is not unreasonable to assume that it is linear with respect to M_{exist} as well.

If we *postulate* that $\widehat{\delta T}$ is a linear operator with respect to M_{exist}, the most general linear transform on $\mathbb{S}^1$ is a rotation by angle $\delta\theta$. Since the local laws of physics, including evolution through local time, should not change if we change the choice of δT, an application of eq. (27) when changing the choice of δT implies that $\delta\theta$ is a linear function of δt, i.e. $\delta\theta = \omega\,\delta t$. Therefore, after applying the time definition in eq. (12), we expect Ψ to take a general form $\Psi(x, y, z, t) = e^{i\omega t}$, and the probability of presence takes the form of a planar wave from a perspective of local time.

This is exactly what is being observed experimentally for example in Young's slits experiment. If we perform a relativistic change of coordinates, we obtain the more general form $\Psi(\vec{x}) = e^{i\langle\vec{k}\,|\vec{x}\,\rangle}$, where $\vec{x}$ is the position 4-vector and $\vec{k}$ is the wave 4-vector. This "planar wave" condition remains true for all universe fragments $\mathfrak{F}$ connected through one another by δT that are indistinguishable from one another for M_{exist} and where $\widehat{\delta T}$ is linear.

4.3 Interpretation

Implications of this parallel between quantum mechanics and the TIM are discussed below.

Disentanglement In quantum mechanics, combining two systems is done by considering the tensor product of the spaces describing each individual system. At the limit, the universe would be represented by a wave-function in an infinite tensor product combining the Hilbert space of all possible systems (a Fock space), and the universe would contain any possible entangled state. An ad-hoc simplification in quantum mechanics is to ignore all but the Hilbert space describing the system, as if the rest of the universe was not there.

We justify this simplification with eq. (3). Measurements are designed to provide focus in the physical realm, and to ignore all but a selected fragment of the universe.

Real eigenvalues In quantum mechanics, the possible results of a measurement are postulated to be the eigenvalue of an observable, i.e. a self-adjoint operator on a Hilbert space. Quantum measurement results are real numbers because observables only have real eigenvalues.

In the TIM, measurement results for any physical measurement are always discrete, because we do not know how to make measurements that are truly continuous. However, if the symbolic values for each measurement value are real numbers, one can build a linear operator $\check{M}$ that has the possible measurement values as real eigenvalues.

Wave-function Collapse Since quantum mechanics is linear and reversible, it is difficult to explain why the wave-function should collapse. The need for this rule is widely considered as a problem, and many suggestions have been made to try and explain it using a wide number of approaches[30, 31, 32], some even referring to concepts like "conscious observers" that are extremely difficult to define precisely.

In the present formalism, the wave-function collapses because the measurement system is designed to produce a repeatable numerical result. The wave-function collapse is not instantaneous but happens as part of the measurement process itself, and is a consequence of the non-linearity of $\hat{M}$. It is possible to consider *incomplete* measurements where the resulting measurement probability $|\psi\rangle$ is only partially collapsed, for example to represent measurements containing a dose of uncertainty, like $2.56V \pm 0.2\%$.

There may still be a minor reference to a conscious observer, but it happens before the experiment. Someone performs a *choice* of experimental setup, a selection of valid measurement processes among all possible physical processes. On the other hand, we do not need to hypothesize that something happens just because a conscious observer *looks at* the experiments.

While the present theory does not need an explicit postulate to predict the collapse of the wave-function, it ultimately does not answer the question "*why are there measurements?*". In a sense, the question has only been rewritten. In the context of the present theory, quantum decoherence[33] for example is a useful mathematical analysis of how measurements might appear as a consequence of more fundamental processes.

Superposition In quantum mechanics, any linear combination of valid states is a valid state. In particular, linear combinations of eigenvectors that are not themselves eigenvectors are acceptable kets.

The present formalism observes that in general, a sequence in time or juxtaposition in space of physically acceptable processes is a physically acceptable process. There are physical processes that are not measurements and do not obey eq. (2) $M\,M\,\mathfrak{F} = M\,\mathfrak{F}$. They do not need to turn any universe fragment into an "eigenstate" of the measurement. Consequently, it is safer to assume that any probability state $|\psi\rangle$ in $\mathbb{S}^{n-1}$ may represent a physically acceptable prediction for M applied to some hypothetical state $\mathfrak{F}$.

The method used to justify the use of a wave-function as a representation of a particle probability also explains why, in Young's slits experiment, *removing* a part of the system (the second slit) would actually cause the wave-functions to *add up* instead of being subtracted. The reason is that by removing part of the obstacle, we actually added a new possibility for the particle to cross over, so we need to add the probabilities of measuring a `found` or `not-found` state.

Incomplete measurements In quantum mechanics, probabilities only exist before a measurement, and are defined by how an "entangled" ket, a linear combination of eigenbasis kets, would project on the eigenbasis. After the measurement, there are no more probabilities, both the measurement result and corresponding ket state are known with absolute precision.

In practice, measurements are not always exact nor instantaneous[34, 35]. The result itself may be probabilistic or contain a dose of uncertainty. The formalism presented here allows the state after a measurement result to be itself a probability vector. There is no need for any "instantaneous collapse" either. As the measurement progresses, the distribution of probabilities becomes narrower, and the set of compatible universe states becomes smaller. At least for some measurements, the fully collapsed wave-function and the infinitely precise measurement need only be idealized limits.

Measurement limits Many quantum physics textbooks state confidently that the probability of finding the particle being observed on the other side of the moon[36] is infinitesimal, but not null. This is a logical consequence of the fact that, in the majority of cases, the mathematical wave-function vanishes quickly with distance but never reaches zero.

In the formalism presented here, only results that can actually be generated by the measurement are acceptable. If your measurement apparatus is a photographic plate measuring 10x10 centimeters, the chances that it will report that it found the photon somewhere on the moon (or anywhere outside the plate, for that matter) is *exactly* zero. The point is not that the probability for the photon to miss the plate is zero, only that we cannot talk about the probability to detect the photon at a specific location where there is no detection instrument.

This may seem like a ridiculously small point, but it actually is the primary experimental proof of the validity of the TIM. Quantum mechanics, as applied using the traditional formalism, routinely predicts precise measurement results even outside of the known limits of the measurement apparatus. The TIM, in contrast, only predicts probabilities for the physically possible outcomes.

Linearity The mathematics of quantum mechanics uses linear algebra on the system states, as represented by wave-functions or kets.

The present formalism represents probability distributions as a vector in $\mathbb{S}^{n-1}$. In general, operators on the probability states are not linear, but for a measurement M giving real measurement results, the linear operator $\check{M}$ has the properties associated to observables in quantum mechanics.

Commutation of Observables For mathematical reasons, quantum mechanics observables that do not commute correspond to measurements that cannot be done independently. Heisenberg's inequality is the archetypal application of that observation for position and momentum observables.

In the present formalism, the reasoning relating eq. (7), eq. (8) and eq. (9) suggest that, as long as a physical process is reversible, independent measurements correspond to physical processes that commute, and conversely, measurements that do not commute impact one another and do not give independent results. Once non-commutativity is established, the quantitative mathematical treatment given to the probability state $|\psi\rangle$ on the tensor product space is identical to the mathematical treatment given in quantum mechanics to the ket $|\psi\rangle$. The very rich set of results obtained by studying commutation relations is therefore preserved.

Quantum space-time In the traditional approach of quantum mechanics, the space and time operators commute with one another, because they are based on multiplying a ket with a coordinate, and the multiplication of real numbers commutes. Based on the above analysis, this defines Euclidean space-time coordinates, and this is clearly a source of conflict both with physical observation and with general relativity.

Research in the field of non-commutative geometry[37] has long been seen as one of the promising paths towards a quantum theory of gravity. However, most applications to physics remain firmly grounded in mathematics[38] more than in considerations of what the mathematical symbols represent. The present work shows a relatively simple physical interpretation for this work. More importantly, it suggests that surprising aspects of it should not be too quickly discarded as mathematically absurd.

For example, one particular issue with respect to a traditional or perturbative treatment of space-time non-commutativity is the appearance of non-unitarity in the S-matrix[39]. Non-unitarity, if interpreted as the sum of all probabilities no longer being 1, raises a serious problem from a quantum mechanics point of view. But if we accept that non-commutativity must be possible, then this indicates that the S-matrix lost its physical significance. This is not surprising considering that a S-matrix relates the state of the system to states "at infinity", and those depend on the large-scale space-time structure. By analogy, if one is computing the probability to find a mobile running on the surface of earth, summing probabilities on a tangent plane will yield incorrect results.

Hidden Variables The well known theorem of Bell gave a verifiable method for determining if quantum mechanics can be explained by hidden variables. While the various kinds of hidden variable theories (local, non-local, stochastic) make a definitive conclusion on this topic harder to reach, to this day, experiments conducted by Alain Aspect [40] and others would tend to disprove hidden variable theories.

The interpretation of quantum theory given here does not rely on hidden variables. The reasoning is based entirely on the existence of probabilities based on what we know, and this is one meaning we gave to *incomplete*. But it does not imply that we can know more than what measurements give us. The existence of randomness in the most precise description we can make of the universe remains possible. Thus, even if the measurement is said to be incomplete, it may still be the most complete description of a system we can give.

5 Conclusion and Further work

We have presented a formalism to discuss physical experiments without implicitly assuming a specific mathematical structure. This allowed us to propose a definition of measurements, and based on formal reasoning, to reconstruct many important aspects of both general relativity and quantum mechanics, while suggesting a number of limitations in these theories which were not obvious from a purely mathematical standpoint. We should now try to understand the impact of these limitations on experiments we have made or will make.

Fundamental equations One important closing remark is that we did not present any "fundamental equation", i.e. something equivalent to the field equation in general relativity or to Schroedinger's equation in quantum mechanics. An equation like eq. (13) is clearly not specific enough to make predictions. As a result, the TIM principles alone are not sufficient to make predictions, and additional relations must be written.

For the moment, we have simply "borrowed" the existing fundamental equations from the respective theories, with an understanding that these may be approximations corresponding to a particular choice of physical measurement. This way, we were able to indirectly leverage the numerous advances made during the past century, including the many forms of Lagrange equations and the gauge invariance arguments that have proven so successful in predicting particle interactions.

In that sense, the present theory is "backwards" from most recent research: whereas recent work is often about suggesting a new fundamental equation, generally by means of a new formulation of the Lagrangian or action, the theory presented here replaces practically everything but the fundamental equations.

Characteristics of the theory The TIM has the following characteristics:

- All measurements are treated identically, including time and space measurements.
- The theory addresses incomplete measurements, i.e. measurements from which only probabilistic measurements can be made. This is true whether the uncertainty is due to a partial collection of data we believe to be complete, or whether the uncertainty itself is believed to be fundamental.
- When measurements results can as far as we know only be predicted statistically, the theory is formally almost identical to quantum mechanics, and therefore incorporates its predictions by reference. In particular, the theory predicts the mathematical shape of the quantum mechanics wave-function under a reasonable set of conditions. Quantum mechanics is seen as an approximation, where non-linearity, divergence at large scale or non-commutativity of space and time operators are ignored. The TIM also offers yet another interpretation of quantum mechanics.
- At large scale, the effects of space and time non-commutativity become dominant, but measurements return quasi-deterministic results, so that measurement values can be approximated by continuously differentiable functions. In that case, the present formalism becomes formally identical to general relativity, and therefore incorporates its predictions by reference. General relativity is also seen as an approximation, where measurements can be identified to their continuous approximation.
- The transition between different time and space measurements introduces an explicit resolution factor λ, which is assimilated to the scale factor formally introduced in Nottale's theory of scale relativity. It is therefore expected that the present theory also incorporates scale relativity results by reference, and offers a physical justification.

Future work Obviously, simply borrowing fundamental equations and incorporating them in the present theory is not entirely satisfying. For this reason, finding a fundamental equation, ideally derived from TIM principles alone, should now be the most active topic of research.

In that respect, we can already make a number of preliminary observations. First, we can note that all theories of physics accepted so far have at least one local and one distant fundamental law (and generally several distant laws corresponding to different interactions):

- The local law describes how the system being studied behaves. In general relativity, the equation $D\ u_\mu = 0$ plays that role, whereas in quantum mechanics, various formulations coexist, starting with Schroedinger's equation.
- The distant law describes how the rest of the universe can be "summarized" as far as the local system is concerned. In general relativity, the field equation plays that role. In quantum mechanics, theories are specified by their Lagrangian or, equivalently, by a Hamiltonian or action.

Consequently, it is fair to say that none of our theories at this point can be considered as fully unified, in the sense that none can derive large-scale laws like the shape of the Hamiltonian from local laws like the local evolution of the wave-function.

Attempting to resolve that issue brings the second observation, which we can make using the gravitational interaction. Mass and energy play two complementary roles in general relativity. Locally, mass is a resistance to change, as shown in eq. (25): the higher the mass, the lesser the impact of external interaction. Globally, mass causes change, as shown in eq. (24): the higher the mass, the bigger the impact in external interaction. The relation $E = h\,\nu$ relates these two aspects, and therefore there seems to be a relation between the wave-particle duality and the local-distant duality.

The "large scale" aspects, which "summarize" the rest of the universe, are by construction containing many more degrees of freedom than the "local" aspects. It is therefore reasonable to expect that a statistical analysis is necessary to be able to derive both local and distant formulations from the same principle. Another observation gives us confidence in this approach. The fundamental law, both for quantum mechanics and for general relativity, centers around energy. We know since Boltzmann that there is a correlation between the number of degrees of freedom and entropy. We also know that for a macroscopic adiabatic (closed) system, there is a direct relation between entropy and energy ($\mathrm{d}U = T\,\mathrm{d}S$). It is therefore not unreasonable to speculate that a future "fundamental equation" can be derived from a formulation of time and energy in terms of statistical selection.

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